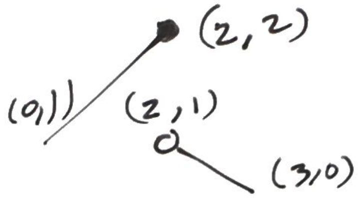
HW 1 − 6 (1.4 p83) Page 1

p 83 51, 52, 53, 55, 59, 63, 65, 67*, 83, 85, 95* (sec 1.4)

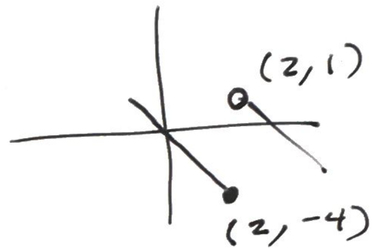
p 92 33*, 37, 39, 41*, 49 (sec 1.5)

(51) $f(x) = \begin{cases} \frac{1}{2}x + 1, & x \le 2 \\ 3 - x, & x > 2 \end{cases}$ find x values where not cont find which are removable



non removable when $x = 2$

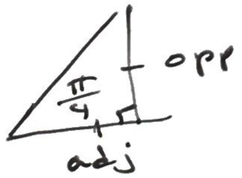
(52) $f(x) = \begin{cases} -2x, & x \le 2 \\ x^2 - 4x + 1, & x > 2 \end{cases}$

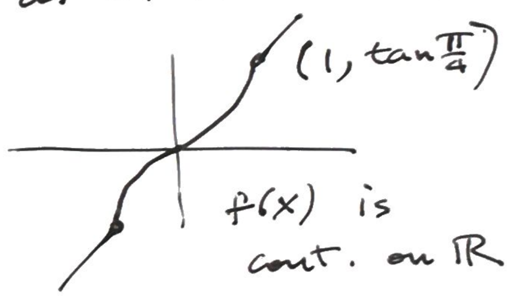


non removable at $x = 2$

(53) $f(x) = \begin{cases} \tan \frac{\pi x}{4}, & |x| < 1 \\ x, & |x| \ge 1 \end{cases}$ ← same as $-1 < x + 1$

Recall isoceles Rt Δ:





$f(x)$ is cont. on ℝ

(55) $f(x) = \csc 2x = \frac{1}{\sin 2x}$

so vert. asym. at $2x = n\pi$, $n \in \mathbb{Z}$

$x = \frac{n\pi}{2}$, $n \in \mathbb{Z}$.

(non removable

Find a for continuity

(59)

$f(x) = \begin{cases} 3x^2, & x \ge 1 \\ ax - 4, & x < 1 \end{cases}$

Since $3x^2 \Big|_{x=1} = 3$

We need $ax - 4 = 3 \Big|_{x=1}$

$a(1) - 4 = 3$

$a = 7$

(63)

$f(x) = \begin{cases} 2, & x \le -1 \\ ax + b, & -1 < x < 3 \\ -2, & x \ge 3 \end{cases} \Rightarrow$ $a(-1) + b = 2$

$a(3) + b = -2$

$4b = 4 \quad b = 1$

$a = b - 2 = -1$

HW 1-6 (1.4 p 83) Page 2

p 83   65, 67*, 83, 85, 95*

(65) $f(x) = x^2$ Discuss cont of $h(x) = f(g(x))$

$g(x) = x - 1$ $h(x) = f(x-1) = (x-1)^2$

Continuous on $\mathbb{R}$ as polynomials are cont. and Th 1.12

(67)* $f(x) = \frac{1}{x-6}$ $h(x) = f(x^2+5) = \frac{1}{(x^2+5)-6}$

$g(x) = x^2+5$ So $x^2+5-6 \neq 0$

$x^2 \neq 1$

x cannot be ± 1

Why must there be a zero on the interval?

(83)

$f(x) = \frac{1}{12}x^4 - x^3 + 4$ on $[1,2]$

Since f is cont. on $[1,2]$ and

$f(1) = 3.08\overline{3}$ and $f(2) = -2\frac{2}{3}$

There is a c in $[1,2]$ such that $f(c) = 0$

by the I.V.T.

(zero is in $[3.08\overline{3}, -2.\overline{6}]$

(85) $f(x) = x^2 - 2 - \cos x$ on $[0, \pi]$

Since $f(x)$ is continuous on $[0, \pi]$,

by the IVT there is a c on $[0, \pi]$

such that $f(0) \leq f(c) \leq f(\pi)$

$f(0) = -3$ and $f(\pi) \approx 8.869$

there is a $f(c) = 0$

(95) $f(x) = x^2 + x - 1$ on $[0,5]$, find c so $f(c) = 11$

Since f is cont on $\mathbb{R}$ the IVT states

there is a c where $f(0) = -1 \leq f(c) = 11 \leq f(5) = 29$

$x^2 + x - 1 = 11$

$x^2 + x - 12 = 0$

$(x-3)(x+4) = 0$

$c = 3$

(Note while -4 makes a zero, it's not in $[0,5]$

HW 1-6 (1.5 p 92) Page 3 ##

p92 33*, 37, 39, 41*, 49

Is the discontinuity @ $x=-1$ removable or a vertical asymptote

(33) $f(x) = \dfrac{x^2-1}{x+1}$ $\quad f(-1) = \dfrac{(-1)^2-1}{-1+1} = \dfrac{0}{0}$ indeterm. form.

$f(x) = \dfrac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}}$ so a hole, a removable discontinuity

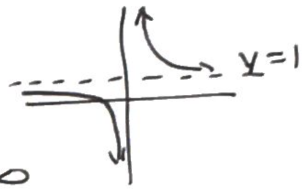
(37) $\displaystyle\lim_{x\to 2^+} \frac{x}{x-2} = \frac{2^+}{2^+-2} = +\infty$

(39) $\displaystyle\lim_{x\to -3^-} \frac{x+3}{x^2+x-6} = \lim_{x\to -3} \frac{(x+3)}{(x+3)(x-2)}$

$\displaystyle = \frac{1}{-3-2} = \frac{-1}{5}$

(41) $\displaystyle\lim_{x\to 0^-} \left(1+\frac{1}{x}\right) = \lim_{x\to 0^-}\left(\frac{x+1}{x}\right) = \frac{0^-+1}{0}$

$= -\infty$



(49) $\displaystyle\lim_{x\to (\frac{1}{2})^-} x \sec(\pi x)$

$\displaystyle\lim_{x\to (\frac{1}{2})^-} \frac{x}{\cos \pi x} = \frac{\frac{1}{2}^-}{\cos(\frac{\pi}{2})^-} = \frac{\frac{1}{2}^-}{0^-} = +\infty$

$\frac{1}{2} > 0$ and left of $\cos\frac{\pi}{2} > 0$

confirm w/ numerical and graphical methods:

x	.49	.449	.5	.501	.51
f(x)	15.6	158.8	?	-159	-16